

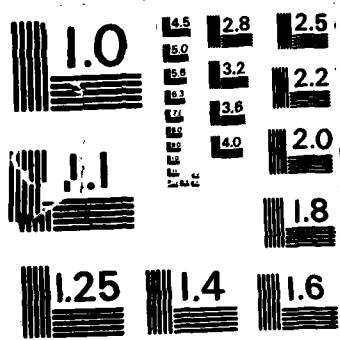
AD-A167 783 PERSPECTIVE PROJECTION INVARIANTS(U) MASSACHUSETTS INST 1/1
OF TECH CAMBRIDGE ARTIFICIAL INTELLIGENCE LAB
A VERRI ET AL FEB 86 AI-M-832 N00014-80-C-0505

UNCLASSIFIED

F/G 12/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

AD-A167 783

UNCLASSIFIED

2

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER A.I. Memo. 832	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Perspective Projective Invariants.		5. TYPE OF REPORT & PERIOD COVERED A.I. Lab. Memo.
7. AUTHOR(s) A. Verri. and A.L.Yuille.		6. PERFORMING ORG. REPORT NUMBER N00014-80-C-0505
9. PERFORMING ORGANIZATION NAME AND ADDRESS Artificial Intelligence Laboratory 545 Technology Square Cambridge, MA 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Advanced Research Projects Agency 1400 Wilson Blvd. Arlington, VA 22209		12. REPORT DATE February, 1986.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Information Systems Arlington, VA 22217		13. NUMBER OF PAGES 16.
16. DISTRIBUTION STATEMENT (of this Report) Distribution is unlimited.		15. SECURITY CLASS. (of this report) Unclassified.
		16a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES None		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Stereo, Registration, Perspective Projection, Zeros of Curvature.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An important part of stereo vision consists of finding and matching points in two images which correspond to the same physical element in the scene. He shows that zeros of curvature of curves are perspective invariants and can therefore be used to find corresponding points. They can be used to help solve the registration problem (Longuet-Higgins 1982) and to obtain the correct depth when a curve enters the forbidden zone. (Krol and van de Grind 1982). These are also relevant to theories for representing image curves. We consider the stability of these zeros of curvature.		

DTIC FILE COPY

DD FORM 1 JAN 73 EDITION OF 1 NOV 68 IS OBSOLETE
S/N 0102-014-66011

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

88 5 21 071

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

and

CENTER FOR BIOLOGICAL INFORMATION PROCESSING
WHITAKER COLLEGE

A.I. Memo 832
C.B.I.P. Paper 011

February, 1986

Perspective Projection Invariants

Alessandro Verri and Alan Yuille

Abstract: An important part of stereo vision consists of finding and matching points in two images which correspond to the same physical element in the scene. We show that zeros of curvature of curves are perspective projection invariants and can therefore be used to find corresponding points. They can be used to help solve the registration problem (Longuet-Higgins, 1982) and to obtain the correct depth when a curve enters the forbidden zone (Krol and van de Grind, 1982). They are also relevant to theories for representing image curves. We consider the stability of these zeros of curvature.

© Massachusetts Institute of Technology, 1986

This report describes research done at the Massachusetts Institute of Technology within the Artificial Intelligence Laboratory and the Center for Biological Information Processing (Whitaker College). Support for the Laboratory's artificial intelligence research is provided in part by the Advanced Research Projects Agency of the Department of Defense under Office of Naval Research contract N00014-80-C-0505. The Center's support is provided in part by the Sloan Foundation and in part by Whitaker College. A. Verri was partially supported by a fellowship from ELSAG S.p.A. Genoa, Italy.

Perspective Projection Invariants

The goal of stereopsis in computer vision is the recovery of the 3D structure of a scene, given two different images of it. This recovery is preceded by the search for corresponding features of the two images, that is to say features that correspond to the same physical elements in the scene (1,17).

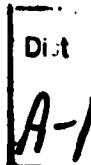
If the two images are taken from two close points of view, they will typically be just slightly different: then standard correlation techniques can identify corresponding elements (2,3,4,5,6,7,8). Once this correspondence has been obtained, triangulation methods allow the 3D reconstruction of the scene. It can be shown that in this case the error made in the reconstruction is large, while if the two images are taken from two very different points of view the error is smaller (9). However when the two images are taken from points of view far apart from each other they will look very different and standard correlation techniques will fail in the search for corresponding elements. This search, therefore, must be guided by new rules, taking into account features having invariant properties under arbitrary perspective projections. Such invariant features will project to corresponding points in the two images.

Epipolar lines in the two images are lines on which corresponding points lie. The projections of a point P in space lie on the plane defined by P and the two camera foci and hence on the two lines defined by the intersection of this plane with the two image planes (see figure 1).

If the epipolar lines are known the matching problem is reduced to a one-dimensional search. Moreover the ordering of edges, or other features, is usually preserved along epipolar lines. Many stereo algorithms make use of the ordering constraint along epipolar lines. This ordering constraint is violated for an object in the "forbidden zone" (14, 15, 16). If a point B lies in the forbidden zone of a point A and the two points are connected by an opaque surface then the two images would see opposite sides of the surface (see figure 2).

To determine the epipolar lines we must find the camera geometry by registering the image. Longuet-Higgins (1982) has described a method for solving the registration problem assuming that three corresponding points in the two images are known. Perspective invariants project to corresponding points in the two images and hence could be used to solve this problem. They could also be used as consistency checks to see if the ordering constraint is violated. When the object enters the forbidden zone the ordering constraint will give false matches. These false matches could be detected if we could find corresponding points.

<input checked="" type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>
y Codes	
Aval and/or Special	



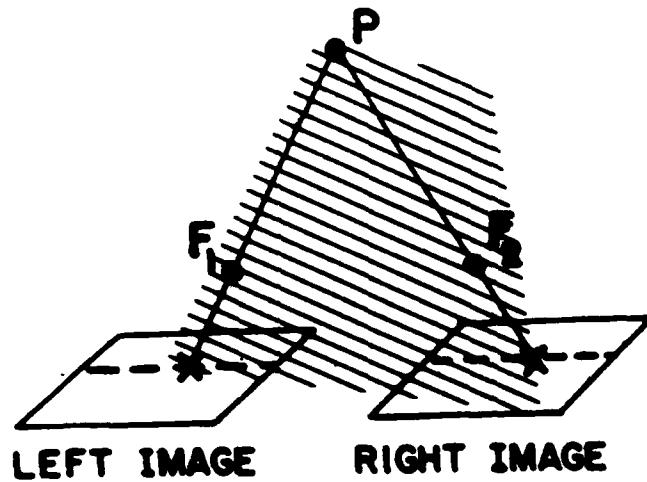


Figure 1. See text.

In this note we prove that points of zero curvature of planar curves are "perspective invariants" and that points of zero curvature of non planar curves are almost always "perspective invariants" with the exception of non generic cases(10). Moreover we briefly analyze the stability in 2D and 3D of such points, considered as zeros of curvature. We show that the zeros of curvature of planar curves are structurally stable for a 2D deformation (one lying on the plane of the curve) but not for a 3D one. Since projections map 3D curves into 2D ones, however, projections of almost planar curves show some stability properties. On the contrary, zeros of curvature of arbitrary 3D curves do not present any simple kind of stability.

Thus zeros of curvature of planar or almost planar curves are features that can be successfully used in solving the "correspondence problem" in stereopsis when the scene is observed from two very different points of view. Since zeros of curvature in two images correspond they can be used to solve the registration problem for stereo (13). The projection invariants of zeros of curvature and the constraint that matches take place between points on corresponding epipolar lines have been used to construct an algorithm able to solve the correspondence problem for arbitrary 2D curves even when the ordering constraint is violated (11).

Recent work on representing image curves involves filtering the curve with gaussians of

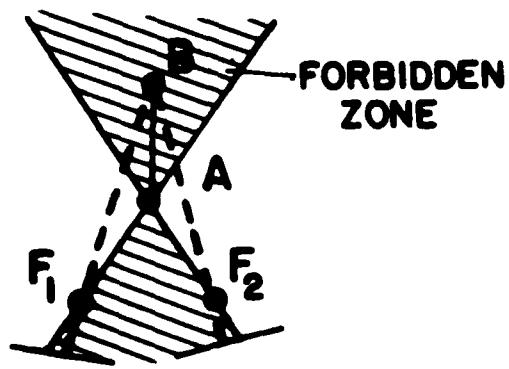


Figure 2. See text.

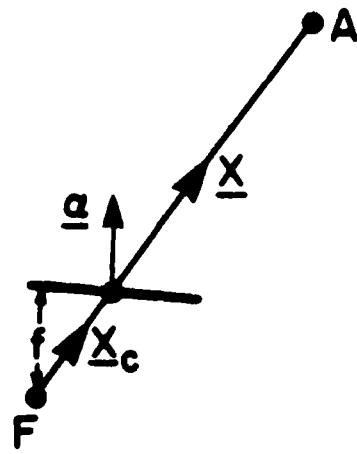
different sizes to obtain a multiscale description which can be interpreted in terms of a small set of primitives thereby producing a *curvature primal sketch* (18). Marimont (19) considers planar curves projected orthographically onto the viewing plane. He suggests using zeros and extrema of curvature as primitives because of their invariance, and approximate invariance, under change of viewpoint. Zeros of curvature of the curve are moreover preserved when the curve is convolved with a gaussian filter. The results in this paper extend this work by showing that zeros of curvature are invariants even when the curve is non-planar and the projection is perspective.

Section 1.

Let γ be a curve in the 3D space and γ_p its perspective projection with respect to a focus O on a plane Φ . Let f be the focal length, the distance between O and Φ .

If we introduce a system of coordinates (x, y, z) centered on O then γ will be described by the vector valued function $x(s) = (x(s), y(s), z(s))$ where s , the arc length of γ , varies in a given interval $[a, b]$ of $I\!\!R$ and γ_p by $x_p = x_p(s)$. It is easy to see that the following relationship holds between x_p and x (see fig.3)

$$x_p(s) = \frac{f}{(x(s) \cdot \omega)} x(s) \quad (1.1)$$

**Figure 3.** See text.

where α is the unit normal vector to the projection plane Φ : notice that $z \cdot \alpha \neq 0$ for all s since we suppose that the perspective projection exists.

From now on $T(T_p)$ will indicate the unit tangent vector to the curve (projected curve), $N(N_p)$ the unit normal to the curve (projected curve) and $B(B_p)$ the unit binormal to the curve (projected curve). Suppose moreover that each component of $z(s)$ is a function twice differentiable with respect to s .

From elementary differential geometry we have:

$$\kappa(s) = \left\| \frac{d^2 z(s)}{ds^2} \right\| \quad (1.2)$$

where $\kappa(s)$ is the curvature of γ at the point $z(s)$ and:

$$\kappa_p(s_p) = \left\| \frac{d^2 z_p(s_p)}{ds_p^2} \right\| \quad (1.3)$$

where $\kappa_p(s_p)$ is the curvature of γ_p at the point $z_p(s_p)$, with s_p being the arc length of γ_p .

Now let us compute κ_p . It is easy to see that:

$$\frac{d\mathbf{x}_p}{ds} = \frac{\mathbf{f}}{(\mathbf{x} \cdot \alpha)^2} \mathbf{L} \quad (1.4)$$

where

$$\mathbf{L} = \alpha \times (\mathbf{T} \times \mathbf{x}). \quad (1.5)$$

Since s_p is the arc length of \mathbf{x}_p ,

$$\mathbf{T}_p \cdot \mathbf{T}_p = \frac{d\mathbf{x}_p}{ds_p} \cdot \frac{d\mathbf{x}_p}{ds_p} = 1 = \frac{d\mathbf{x}_p}{ds} \cdot \frac{d\mathbf{x}_p}{ds} \left(\frac{ds}{ds_p} \right)^2 \quad (1.6)$$

and hence

$$\frac{ds}{ds_p} = \frac{(\mathbf{x} \cdot \alpha)^2}{\mathbf{f}} \frac{1}{|\mathbf{L}|}. \quad (1.7)$$

Notice that (1.7) holds if

$$|\mathbf{L}| \neq 0 \quad (1.8a)$$

and that (1.8a) always holds (see (1.3)) unless¹

$$\mathbf{T} \parallel \mathbf{x}. \quad (1.8b)$$

Now

$$\mathbf{T}_p = \frac{\mathbf{L}}{|\mathbf{L}|} \quad (1.9)$$

and

$$\kappa_p \mathbf{N}_p = \frac{d\mathbf{T}_p}{ds_p} = \frac{d\mathbf{T}_p}{ds} \frac{ds}{ds_p}. \quad (1.10)$$

Substituting (1.9), (1.7) and (1.3) in (1.10) gives

$$\kappa_p \mathbf{N}_p = \frac{d}{ds} \left(\frac{\mathbf{L}}{|\mathbf{L}|} \right) \frac{(\mathbf{x} \cdot \alpha)^2}{\mathbf{f}} \frac{1}{|\mathbf{L}|} \quad (1.11)$$

but

¹If this condition is not satisfied then a cusp will typically be generated in γ_p .

$$\frac{d}{ds} \left(\frac{\mathbf{L}}{|\mathbf{L}|} \right) = \frac{1}{|\mathbf{L}|^3} \mathbf{L} \times \left(\frac{d\mathbf{L}}{ds} \times \mathbf{L} \right). \quad (1.12)$$

and so

$$\kappa_p \mathbf{N}_p = \frac{(\mathbf{z} \cdot \mathbf{\alpha})^2}{f|\mathbf{L}|^4} \mathbf{L} \times \left(\frac{d\mathbf{L}}{ds} \times \mathbf{L} \right). \quad (1.13)$$

Notice that

$$\frac{d\mathbf{L}}{ds} = \kappa \mathbf{\alpha} \times (\mathbf{N} \times \mathbf{z}) = \kappa \mathbf{L} \quad (1.14)$$

where we set

$$\mathbf{L} = \mathbf{\alpha} \times (\mathbf{N} \times \mathbf{z}). \quad (1.15)$$

So (1.13) becomes

$$\kappa_p \mathbf{N}_p = \frac{(\mathbf{z} \cdot \mathbf{\alpha})^2}{f|\mathbf{L}|^4} \kappa \mathbf{L} \quad (1.16)$$

where we set

$$\mathbf{L} = \mathbf{\alpha} \times (\mathbf{N} \times \mathbf{z}). \quad (1.17)$$

Notice that $|\mathbf{L}| = 0$ if and only if $\mathbf{L} = \beta \mathbf{L}$ for some real number β . This can easily be seen from (1.17): looking at (1.5) and (1.14) it is obvious that such a β exists if and only if $\mathbf{z}, \mathbf{N}, \mathbf{L}$ are coplanar, that is if and only if $\mathbf{z} \cdot \mathbf{N} \times \mathbf{L} = 0$ (we ignore the possibility that $\mathbf{z} \cdot \mathbf{z} = 0$ since in this case the eye cannot see the point).

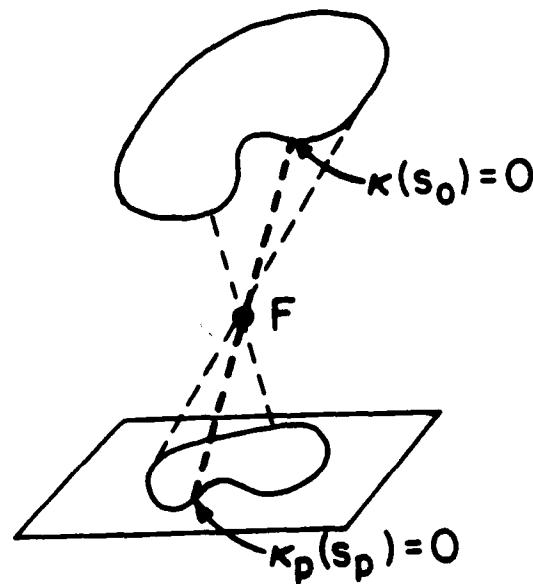
Now we can prove the following theorems.

Theorem 1.

Let γ be a simple planar curve and γ_p its perspective projection onto a plane Φ with respect to a focus O . Let us use the same notation and hypotheses as above. Then: for every s for which κ is defined κ_p is also defined. Moreover, whenever there exists an s_0 such that $\kappa(s_0) = 0$ then $\kappa_p(s_0) = 0$ and vice versa.

Proof

Notice that for a planar curve for which a perspective projection with respect to a focus O exists, $\mathbf{z} \cdot \mathbf{N} \times \mathbf{T} \neq 0$ for all s , since the plane containing the curve does not pass through

**Figure 4.** See text.

the point O. Hence (1.8a) and (1.8b) always hold, thus (1.16) holds as well and the first statement of the theorem is proved.

Moreover from (1.16) it follows immediately that when there exists an s_0 such that $\kappa(s_0) = 0$ then $\kappa_p(s_0) = 0$. The converse is also true since in the right hand side of (1.16) $(\underline{z} \cdot \underline{\alpha})^2 > 0$ for all s by hypotheses and $|\underline{M}| \neq 0$ because as mentioned above $\underline{z}, \underline{\alpha}, \underline{T}$ cannot be coplanar. So if $\kappa_p(s_0) = 0$ (1.17) holds if $\kappa(s_0) = 0$.

Theorem 2

Let us suppose that γ is a generic non planar curve and γ_p its perspective projection with respect to a focus O on a plane Φ . Then with the same notation and hypotheses as above: for every s for which κ is defined and unless $\underline{T} \parallel \underline{z}$ then κ_p is defined. Moreover whenever there exists s_0 such that $\kappa(s_0) = 0$ then $\kappa_p(s_0) = 0$ (see fig.4).

Proof

Notice that for any curve (1.8a) holds if (1.8b) holds, that is it holds unless $\underline{T} \parallel \underline{z}$. Then from the hypotheses of the theorem the proof follows directly from (1.16).

Theorem 3

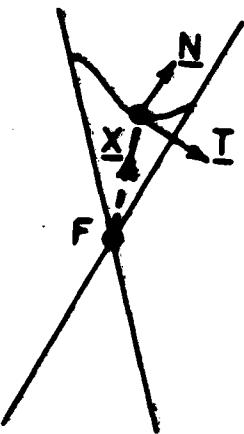


Figure 5. See text.

With the same hypotheses as theorem 2 a necessary and sufficient condition for there to exist an s_0 such $\kappa_p(s_0) = 0$ while $\kappa(s_0) \neq 0$ is that $z(s_0) \cdot N(s_0) \times T(s_0) = 0$.

Proof

The theorem is a direct consequence of the fact that, by the hypotheses of theorem 2, (1.16) holds and that $|M(s_0)| = 0$ if and only if $z(s_0)$, $N(s_0)$ and $T(s_0)$ are coplanar.

Theorem 1 says that for any planar curve points of zero curvature are preserved under arbitrary perspective projection: theorem 2 says that for arbitrary curves points of zero curvature are transformed into points of zero curvature of the projected curve. The converse, as shown in theorem 3, is true except for the case in which z , N and T are coplanar (see fig.5).

This case, however, is non generic since a small displacement of the focus of projection will destroy the coplanarity.

It is worth noticing that even cusps have the same property of invariance, but as discontinuities of the curvature they are not easily identified in computer implementation (while zeros of curvature are).

Section 2.

In this section we discuss the stability of 2D and 3D zeros of curvature. Before starting the discussion let us summarize some elementary results of differential geometry (12).

Let γ be a smooth curve lying on a smooth surface. The component of the curvature vector $d^2\mathbf{z}/ds^2$ along the direction of the surface normal is called the normal curvature. Therefore if \mathbf{n} is the unit vector normal to the surface and κ_n the normal curvature, then for every s we have:

$$\kappa_n = \frac{d^2\mathbf{z}}{ds^2} \cdot \mathbf{n} = \kappa \mathbf{N} \cdot \mathbf{n}. \quad (2.1)$$

Moreover the component of the curvature vector along the unit vector \mathbf{u} , orthogonal to the surface normal and such that $\mathbf{u} \cdot (\mathbf{n} \times \mathbf{N}) = 0$ is called the geodesic curvature. It is easy to see that the geodesic curvature at a point is equal to the ordinary curvature times the cosine of the angle θ between the osculating plane to the curve (the plane through the point containing the tangent and normal vectors to the curve) and the tangent plane to the surface. Therefore, if κ_g is the geodesic curvature, for every s we have:

$$\kappa_g = \kappa \cos \theta \quad (2.2)$$

and clearly:

$$\kappa^2 = \kappa_n^2 + \kappa_g^2. \quad (2.3)$$

It is clear that κ_n must have a maximum κ_1 and a minimum κ_2 , called the first and second principal curvatures respectively, since it is a continuous function on a closed bounded set. Moreover the directions of the two principal curvatures are mutually orthogonal and if ϕ is the angle between the tangent to the curve and the first principal curvature we have:

$$\kappa_n = \kappa_1 \cos^2 \phi + \kappa_2 \sin^2 \phi. \quad (2.4)$$

This relationship will be useful later on, since it suggests examples of surfaces on which no curve can show zeros of κ_n and therefore no zeros of curvature at all.

Consider now the set Γ' of curves in the x, y plane with only one zero of curvature. It can be defined as follows: $\gamma \in \Gamma'$ if its parametric equations $x = x(s)$, $y = y(s)$ are continuous with derivatives continuous up to every order; moreover

$$x'^2(s) + y'^2(s) = 1 \quad (2.5)$$

for every s and

$$x''^2 + y''^2 \neq 0 \quad (2.6)$$

for every $s \neq s_0$, where s_0 is the point of zero curvature.

At the point $(x(s_0), y(s_0))$ γ has a zero of curvature and in consequence the normal and the binormal to that curve at that point are not well defined. For a planar curve the normal vector to a curve can be defined to be the unit vector lying in the plane perpendicular to the tangent. Since the functions x and y have continuous second derivatives the left and right limits of the unit normal as s goes to s_0 must exist, for when $s = s_0$:

$$\underline{N} = \left(\frac{y''(s)}{\sqrt{x''^2(s) + y''^2(s)}}, \frac{-x''(s)}{\sqrt{x''^2(s) + y''^2(s)}} \right). \quad (2.7)$$

Moreover these two limits, not necessarily coincident, allow the existence of the following vector product limits:

$$\lim_{s \rightarrow s_0+} \underline{T}(s) \times \underline{N}(s) = \underline{B}(s_0)_+$$

$$\lim_{s \rightarrow s_0-} \underline{T}(s) \times \underline{N}(s) = \underline{B}(s_0)_- \quad (2.8)$$

since

$$\underline{B}(s) = \underline{T}(s) \times \underline{N}(s). \quad (2.9)$$

But

$$\underline{T}(s) \cdot \underline{N}(s) = 0 \quad (2.10)$$

for every $s \neq s_0$ and so

$$\underline{B}(s_0)_+ \cdot \underline{B}(s_0)_- = \pm 1 \quad (2.10)$$

or equivalently

$$\underline{N}(s_0)_+ \cdot \underline{N}(s_0)_- = \pm 1 \quad (2.11)$$

where $\underline{N}(s_0)_+$ and $\underline{N}(s_0)_-$ are respectively the right and left limits of the unit normal for $s \rightarrow s_0$.

Expression (2.10) is more useful than (2.11), since for a planar curve the direction of the binormal is constant. So at such points the limit of the binormal can change sign and therefore we can arbitrarily set the value of the sign of the curvature at a given point where $\kappa \neq 0$ and then detect and define a change of sign whenever the binormal, or rather its limit expression, changes its sign.

We now make use of Morse theory (10). Intuitively if the zero of a function $f(z)$ is a zero crossing (in other words if the function changes sign at the zero) then a small perturbation of the function will not alter the existence of such a zero, although it may move its position. The zero is then said to be stable. Moreover most zeros of functions are zero crossings. This can be contrasted with the behaviour of a function like $g(x) = x^4$ which has a zero, but not a zero crossing, at $x = 0$. A small perturbation of $g(x)$, such as adding an infinitesimal number ϵ will destroy this zero. Thus the typical zeros of a typical function will be zero-crossings and stable under perturbations of the function. In Morse theory these intuitions are made precise: typical functions are Morse functions and have transversal zero-crossings which have structurally stable intersections with the zero axis. For our purposes this result means that zeros of curvature are stable if they are zero-crossings.

Note that if we had adopted the definition of curvature as a positive function there would have been a problem of defining transversality of the function itself with respect to zero.

We now consider three dimensional perturbations of non-planar curves. We will consider curves which lie on a cylinder with a fixed cylindrical projection onto the x, y plane (see fig.6).

Varying the radius of the cylinder gives us a one-parameter family of non-planar curves. Small changes of the radius give rise to small non-planar perturbations of the curve.

Let γ be a planar curve whose parametric equations are $x = x(t)$, $y = y(t)$, where t is the arc length. If we think of every point in the x, y plane as being the projection of a point lying on a cylinder of radius R whose principal axis has equation:

$$x = 0, \quad y = y, \quad z = R \quad (2.12)$$

then the equations of the curve γ_c whose cylindrical projection on the x, y plane gives γ are (see fig.4):

$$x_c = R \sin \frac{x(t)}{R} \quad (2.13)$$

$$y_c = y \quad (2.13)$$

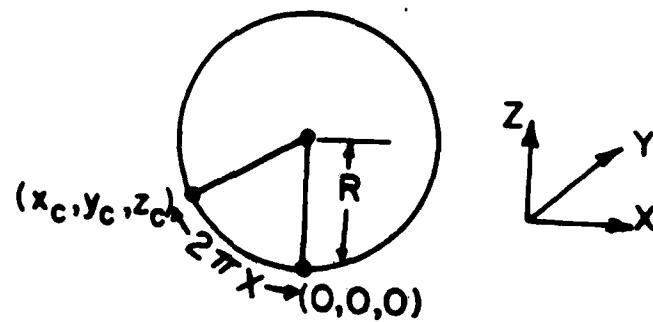


Figure 6. See text.

$$z_c = R - R \cos \frac{x(t)}{R}.$$

Notice that when $R \mapsto +\infty$ the first order approximation of γ_c gives:

$$z_c = z(t)$$

$$y_c = y(t) \quad (2.14)$$

$$z_c = 0.$$

Therefore when R is large, γ_c can be seen as a slight 3D perturbation of γ . Let us now compute κ_c , the curvature of γ_c . Differentiating (2.13) gives:

$$x'_c = \cos \frac{x(t)}{R} x'(t)$$

$$y'_c = y'(t) \quad (2.15)$$

$$z'_c = \sin \frac{x(t)}{R} x'(t).$$

Notice that t is the arc length for γ_c too, since $x'^2 + y'^2 + z'^2 = 1$. Differentiating (2.15) we obtain:

$$x''_c = -\sin \frac{x(t)}{R} \frac{x'^2(t)}{R} + \cos \frac{x(t)}{R} x''(t)$$

$$y''_c = y''(t) \quad (2.16)$$

$$z''_c = \cos \frac{x(t)}{R} \frac{x'^2(t)}{R} + \sin \frac{x(t)}{R} x''(t)$$

therefore

$$\kappa_c^2 = \frac{x'^4(t)}{R^2} + x''^2(t) + y''^2(t). \quad (2.17)$$

It is easy to show that (in the obvious notation)

$$\kappa_{c,n} = \frac{x'^2(t)}{R} \quad (2.18)$$

and

$$x''^2(t) + y''^2(t) = \kappa_{c,g}^2. \quad (2.19)$$

Therefore when $\kappa = 0$, $\kappa_c = 0$ if and only if $x' = 0$, i.e. if the tangent to the curve at the zero of curvature is parallel to the cylindric axis. This condition is usually not satisfied, so 2D zeros of curvature disappear for 3D perturbation.

Despite the instability of the 2D zeros of curvature for 3D perturbations, since projection is a mapping from \mathbb{R}^3 to \mathbb{R}^2 , we can recover in a rough sense some kind of stability for 2D zeros of curvature. In fact every projection of a small 3D perturbation of a planar curve, which had a zero of curvature, shows a zero-crossing of curvature in the neighbourhood of the corresponding projected point. The size of the neighbourhood will depend on the deviation of the surface containing the curve from a plane.

For a generic 3D curve the situation is more complicated since it is not possible to extend the definition of sign of curvature given above. As an example consider a closed smooth curve lying on a cylinder such that there is only one point with a zero of curvature (a curve similar to a distorted sine wave pattern turned in on itself). Explicit computations in this case show that the left and right limits of the binormal at that point have the same direction but opposite sign: however, to define a change of the sign of curvature here leads to a contradictory result since there is a unique such point, by hypothesis, on the curve.

Moreover it is worth noticing that zeros of curvature of 3D curves are extremely uncommon points: in fact looking at the equations it is clear that $\kappa = 0$ if $\kappa_1 = 0$ and $\kappa_1/\kappa_2 = -\tan^2\phi$. That is if, given the surface containing the curve, the ratio of the two principal curvatures assumes a specific value at a specific point.

Summary

We have shown that zeros of curvature of all planar curves and of almost all non-planar curves are perspective invariants. The zeros of curvature of planar curves are shown to be structurally stable for deformations of the curve in the plane. This property still holds for small deformations out of the plane but we prove that it vanishes for arbitrary perturbations of the curve. This lack of structural stability puts a limit on the usefulness of these invariants for non-planar curves. Note, however, this is stability for the curve itself: if the curve does have zeros they will be projective invariants.

Zero crossings of the curvature can be used to find corresponding points for stereo matching. Their use is limited since there are, in general, very few of them. They can be best used to help register the image and to detect if an object enters the forbidden zone.

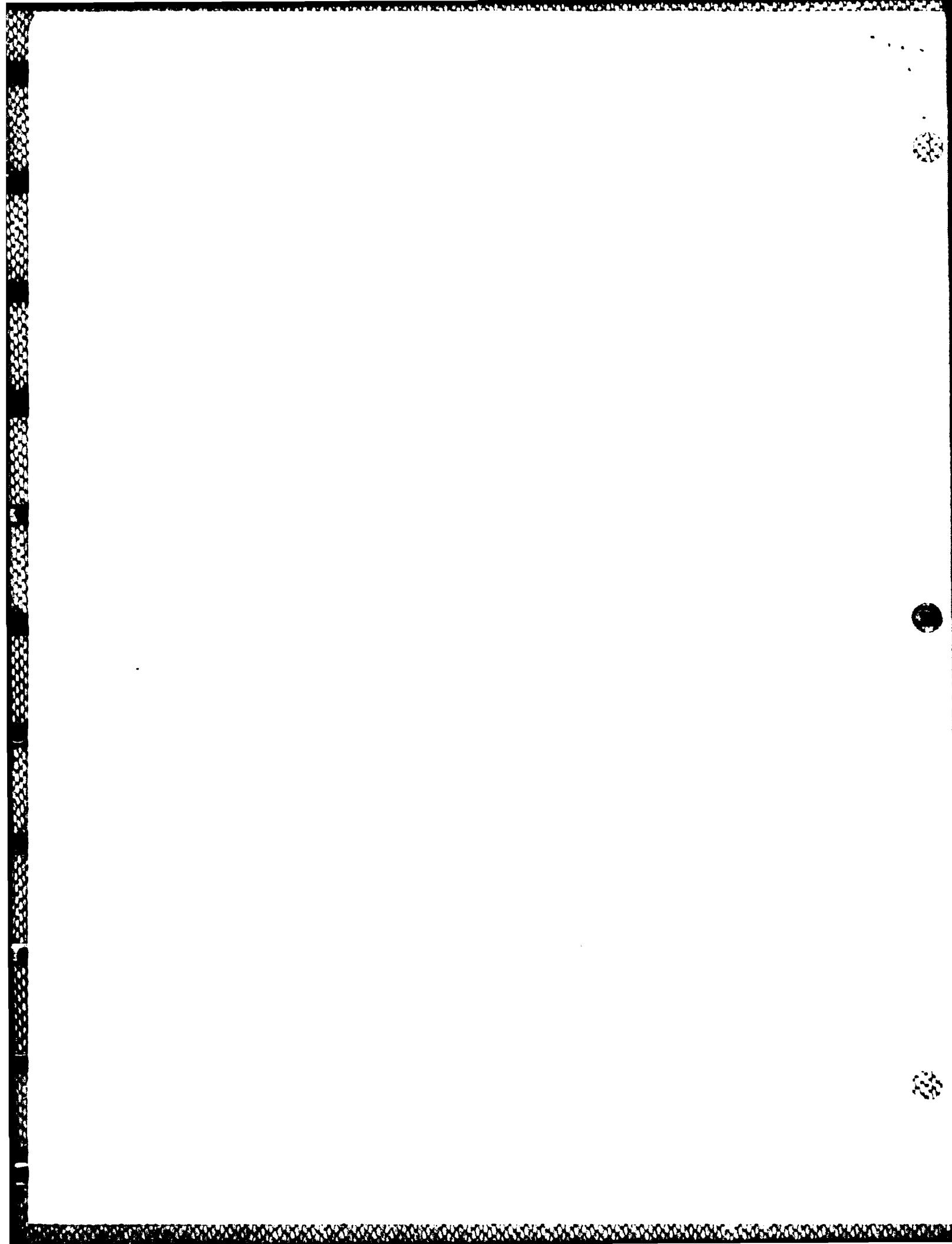
Finally zero crossings are useful for representing image curves because of their invariance under both perspective projection and convolution with gaussian filters.

Acknowledgements We would like to acknowledge helpful conversations with Tommy Poggio and Vincent Torre. Eric Grimson made useful comments on drafts of the paper.

References

- (1) Marr, D., Poggio, T. 1979. "A computational theory of human stereo vision" PROC. ROY. SOC. LOND. B204, 301-328.
- (2) Levine, M.D., Douglas, A., O'Handley, G.M. Yagi 1973. "Computer determination of depth maps" Computer Graphics and Image Processing, 2, 131-150.
- (3) Mori, K-I., Masatsugu, K., Asada, H. 1973. "An iterative prediction and correlation method for automatic stereo comparison" Computer Graphics and Image Processing, 2, 393-401.

- (4) Hannah, M.J. 1974 "Computer matching of area in stereo images" Ph.D. thesis, Stanford Artificial Intelligence Laboratory, AIM-239.
- (5) Panton, D.J. 1978 "A flexible approach to digital stereo mapping" Photogrammetric Engineering and Remote Sensing, vol 44, No 12, dec 1499-1512.
- (6) Moravec, H.P. 1980 "Obstacle avoidance in the real world by a seeing robot rover" Ph.D. thesis, Stanford Artificial Intelligence Laboratory, AIM-340.
- (7) Gennery, D.S. 1980 "Modelling the environment of an exploring vehicle by means of stereo vision" Ph.D. thesis, Stanford Artificial Intelligence Laboratory, AIM-339.
- (8) Nisihara, K. 1983 "PRISM: A practical real-time imaging stereo matcher" in: Intelligent robots: 3th International Conference on Robot Vision and Sensory Controls. Society of Photo-Optical Instrumentation Engineers, Proceedings 449, Cambridge, MA.
- (9) Torre, V., Verri, A. "On the absolute depth estimate in stereopsis" in preparation.
- (10) Poston, T., Stewart, I.N.: "Taylor's expansions and catastrophes" Pitman Publishing London, 1976.
- (11) Sandini, G., Straforini, M., Torre, V., Verri, A. 1984 "3D reconstruction of silhouettes" Proceedings of the 4th International Conference on Robot Vision and Sensory Controls, London.
- (12) Laughwitz, D.: "Differential and Riemannian Geometry" Academic Press, New York 1965.
- (13) Longuet-Higgins, H.C. 1982. "The role of the vertical dimension in stereoscopic vision." Perception 11. 377-379.
- (14) Krol, J.D. and van der Grind, W.A. 1982. "Rehabilitation of a classical notion of Panum's fusional area." Perception, 11. 615-619.
- (15) Verri, A. 1984. "Methodi Matematici per la Visione Stereografica." Ph.D. thesis, University of Genoa.
- (16) Yuille, A.L. and Poggio, T. 1984. "A generalized ordering constraint for stereo correspondence." MIT, A.I. Memo 777.
- (17) Grimson, W.E.L. 1981. "From Images to Surfaces." MIT Press, Cambridge, Mass.
- (18) Asada, H. and Brady, J.M. 1984. "The Curvature Primal Sketch." Proc. IEEE workshop on Computer Vision. Annapolis, Maryland, 1984.
- (19) Marimont, D.H. 1984. "A Representation for Image Curves." Proc. AAAI-84. Austin, Texas.



E N D

Dtic

6 — 86